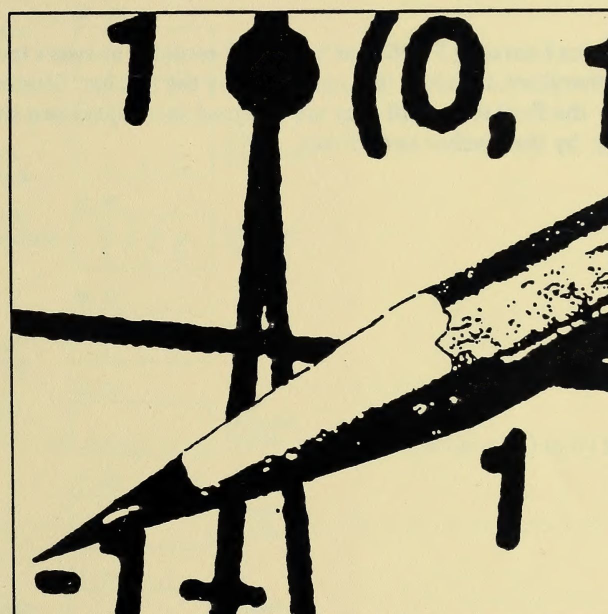
MATHEMATICS 31

LEARNING FACILITATOR'S MANUAL



UNIT 6: ALGEBRAIC VECTORS AND THEIR APPLICATION

Distance Learning

Alberta EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

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Learning Facilitator's Manual
Unit 6
Algebraic Vectors and Their Application
Alberta Distance Learning Centre
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Topic 1: Length of a Vector

2

1. Find the distance between the points $A(3,-2)$ and $B(-1,-3)$.

$$\begin{aligned} d &= \sqrt{(3+1)^2+(-2+3)^2} \\ &= \sqrt{16+1} \\ &= \sqrt{17} \end{aligned}$$

3

2. Find the length of the vector represented by the line segment from $M(3,-2,1)$ to $N(5,0,6)$.

$$\begin{aligned} d &= \sqrt{(5-3)^2+(0+2)^2+(6-1)^2} \\ &= \sqrt{4+4+25} \\ &= \sqrt{33} \end{aligned}$$

5

3. Determine if the three points $P(0,1,0)$, $Q(2,3,1)$, and $R(4,5,2)$ are collinear.

$$\begin{aligned} PQ &= \sqrt{(2-0)^2+(3-1)^2+(1-0)^2} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

$$\begin{aligned} QR &= \sqrt{(4-2)^2+(5-3)^2+(2-1)^2} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

$$\begin{aligned} PR &= \sqrt{(4-0)^2+(5-1)^2+(2-0)^2} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

$$PR = PQ + QR$$

Therefore, P, Q, and R are collinear.

5

4. Determine if the three points $A(3,5)$, $B(0,4)$, and $C(1,1)$ form a right triangle.

$$\begin{aligned} AB &= \sqrt{(3-0)^2+(5-4)^2} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(1-0)^2+(1-4)^2} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(3-1)^2+(5-1)^2} \\ &= \sqrt{20} \end{aligned}$$

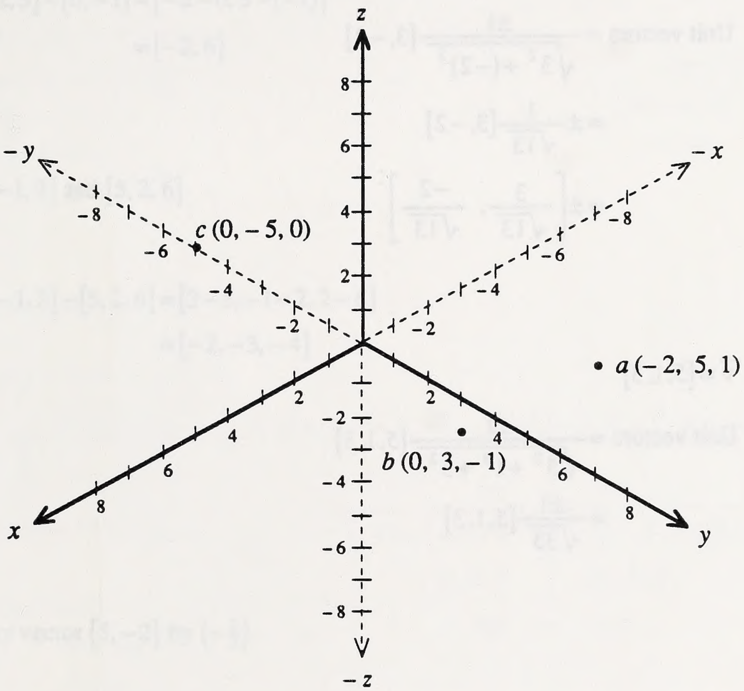
Since $AC^2 = AB^2 + BC^2$, ΔABC is a right triangle.

3

5. Plot the following points.

a. $(-2, 5, 1)$ b. $(0, 3, -1)$

c. $(0, -5, 0)$



2

6. a. The characteristic of a point on the xz-plane is $\underline{\quad y = 0 \quad}$.

b. The characteristic of a point on the z-axis is $\underline{\quad x = y = 0 \quad}$.

Topic 1

_____ marks

Topic 2: Operations Defined on Algebraic Vectors

4

1. Find two unit vectors collinear with each of the following vectors.

a. $\vec{v}=[3,-2]$

$$\begin{aligned}\text{Unit vectors} &= \frac{\pm 1}{\sqrt{3^2+(-2)^2}}[3,-2]\\ &= \pm\frac{1}{\sqrt{13}}[3,-2]\\ &= \pm\left[\frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}\right]\end{aligned}$$

b. $\vec{v}=[5,1,3]$

$$\begin{aligned}\text{Unit vectors} &= \frac{\pm 1}{\sqrt{5^2+1^2+3^2}}[5,1,3]\\ &= \frac{\pm 1}{\sqrt{35}}[5,1,3]\end{aligned}$$

4

2. Find the sums of the following vectors.

a. $[-2,5]$ and $[3,-7]$

$$\begin{aligned}[-2,5]+[3,-7] &= [-2+3,5-7]\\ &= [1,-2]\end{aligned}$$

b. $[0,5,1]$ and $[3,2,-8]$

$$\begin{aligned}[0,5,1]+[3,2,-8] &= [0+3,5+2,1-8]\\ &= [3,7,-7]\end{aligned}$$

4

3. Subtract the second vector from the first vector.

a. $[-2,5]$ and $[0,-1]$

$$\begin{aligned}[-2,5]-[0,-1]&=[-2-0,5-(-1)]\\&=[-2,6]\end{aligned}$$

b. $[3,-1,2]$ and $[5,2,6]$

$$\begin{aligned}[3,-1,2]-[5,2,6]&=[3-5,-1-2,2-6]\\&=[-2,-3,-4]\end{aligned}$$

2

4. Multiply vector $[5,-2]$ by $\left(-\frac{2}{3}\right)$.

$$\left(-\frac{2}{3}\right)[5,-2]=\left[-\frac{10}{3},\frac{4}{3}\right]$$

5

5. Find x and y if $5[x, 3]+2[-3, y]=[4, 1]$.

$$5x-6=4$$
$$5x=10$$
$$x=2$$

$$15+2y=1$$
$$2y=-14$$
$$y=-7$$

5

6. Prove that $\{[0, 5]+[-3, 1]\}+[-5, -2]=[0, 5]+\{[-3, 1]+[-5, -2]\}$.

LS	RS
$[-3, 6]+[-5, -2]$	$[0, 5]+[-8, -1]$
$[-8, 4]$	$[-8, 4]$

LS = RS

6

7. Determine x, y, and z if $2[3, y, 0]-5[x, -2, 2]+[1, 3, -4]=[y, z, x]$.

$$6-5x+1=y$$
$$5x+y-7=0 \qquad (1)$$

$$2y+10+3=z$$
$$2y-z+13=0 \qquad (2)$$

$$0-10+(-4)=x$$
$$x=-14 \qquad (3)$$

Substitute (3) in (1).

$$5(-14)+y-7=0$$
$$y=77$$

Substitute $y=77$ in (2).

$$2(77)-z+13=0$$
$$z=154+13$$
$$=167$$

Topic 2

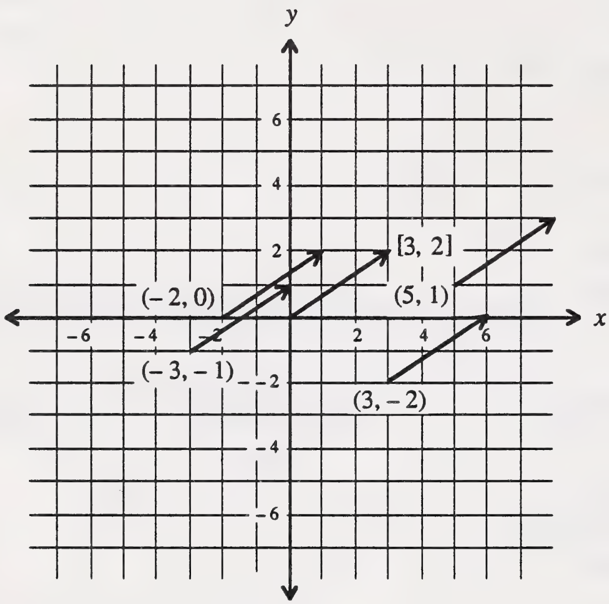
______ marks

Topic 3: Two- and Three-Dimensional Vectors in Algebraic Form

(4) 1. If the position (basic) vector is $[3,2]$, draw an equivalence class of vectors with the following initial points.

a. $(5,1)$ b. $(-2,0)$

c. $(-3,-1)$ d. $(3,-2)$



(2) 2. Find the position (basic) vector which corresponds to the geometric vector $\overrightarrow{AB}$ determined by $A(3,7)$ and $B(5,-2)$.

The coordinates of this position (basic) vector are $[5-3,-2-7]=[2,-9]$.

2

3. Find the coordinates of the position (basic) vector which corresponds to the geometric vector $\overrightarrow{xy}$ determined by x which is at $(3, 2, -5)$ and y which is at $(-1, 0, -2)$.

The coordinates of this position (basic) vector are $[-1-3, 0-2, -2+5] = [-4, -2, 3]$.

2

4. $[3, -1]$ is a position (basic) vector. Find the terminal point of vector $\overrightarrow{PQ}$ which is equivalent to the position (basic) vector. The initial point P is $(4, 5)$.

The terminal point is $(4+3, -1+5) = (7, 4)$.

Topic 3

_____ marks

Topic 4: Collinear and Coplanar Algebraic Vectors

2

1. Find the set of vectors which is parallel to each of the following:

a. $[-5, 1]$

$\{k[-5,1] \mid k \in R\}$

b. $[3, 7, -2]$

$\{k[3,7,-2] \mid k \in R\}$

4

2. Find the set of vectors which is parallel to vector $\overrightarrow{PQ}$ if P and Q are the following points:

a. $P(3,-2) \quad Q(6,4)$

$\overrightarrow{PQ} = [6-3, 4+2] = [3, 6]$

Thus, the set of vectors is $\{k[3,6] \mid k \in R\}$.

b. $P(3,5,-1) \quad Q(0,2,4)$

$\overrightarrow{PQ} = [0-3, 2-5, 4+1] = [-3, -3, 5]$

Thus, the set of vectors is $\{k[-3,-3,5] \mid k \in R\}$.

4 3. $[4, 3, -6]$ and $\left[2, \frac{3}{2}, -3\right]$ are two vectors. Are they collinear? Explain.

Yes, they are collinear because $[4, 3, -6] = 2\left[2, \frac{3}{2}, -3\right]$. They are scalar multiples of each other.

8 4. $[3, 5, k]$ and $[-6, l, 4]$ are two collinear vectors. Find k and l.

$$\begin{aligned} -6 &= (-2)(3) \\ [-6, l, 4] &= (-2)[3, 5, k] \\ &= [-6, -10, -2k] \end{aligned}$$

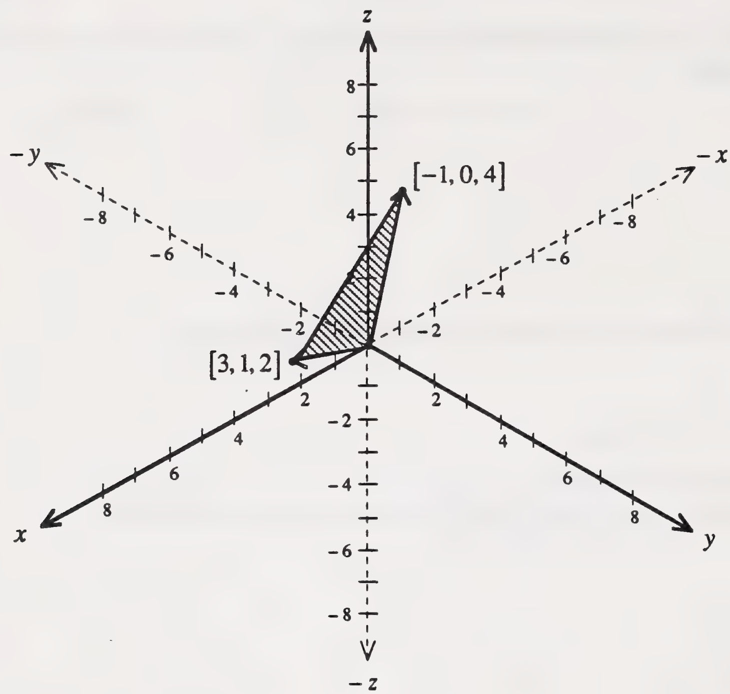
$\therefore l = -10$

$$\begin{aligned} -2k &= 4 \\ \therefore k &= -2 \end{aligned}$$

2 5. $\{k[-5, 0, -3] \mid k \in R\}$ determines the line in three-space through the origin. Find the coordinates of two points on the line.

Answers will vary.

(4) 6. The two position (basic) vectors $\vec{u} = [3,\ 1,\ 2]$ and $\vec{v} = [-1,\ 0,\ 4]$ determine a plane through the origin. Sketch the plane. (Shade the portion of the plane between the vectors.)



(3) 7. Are the vectors $[3,-5]$, $[9,-15]$, and $[-2,7]$ collinear or coplanar?

$[9,-15]=3[3,-5]$

These two vectors are collinear, but $[-2,7]$ is not collinear with either of these. Noncollinear vectors in V_2 are coplanar.

7

8. Are the vectors $[-1,-2,-3]$, $[0,2,1]$, and $[-2,4,-2]$ collinear, coplanar, or noncoplanar?

No two vectors are collinear.

$$\begin{aligned} \text{Let } [-2,4,-2] &= k[-1,-2,-3]+l[0,2,1] \\ &= [-k,-2k,-3k]+[0,2l,\ l] \end{aligned}$$

$$\begin{aligned} -k &= -2 \\ k &= 2 \end{aligned}$$

$$\begin{aligned} -2k+2l &= 4 \\ -4+2l &= 4 \\ 2l &= 8 \\ \therefore l &= 4 \end{aligned}$$

Substitute $k=2$ and $l=4$ in $-3k+l=-2$ as a check.

$$\begin{aligned} \text{LS} &= -3(2)+4 \\ &= -2 \\ \therefore \text{LS} &= \text{RS} \end{aligned}$$

Thus, $[-2,4,-2]$ is a linear combination of the other two. They are coplanar.

6

9. Are the vectors $[3, 5, 7]$, $[0, -2, 1]$, and $[-1, 1, 3]$ collinear, coplanar, or noncoplanar?

By inspecting the vectors, no two vectors are collinear.

Let $[3, 5, 7] = k[0, -2, 1] + l[-1, 1, 3]$.

$$= [0, -2k, k] + [-l, l, 3l]$$

$$3 = 0 - l \quad (1)$$
$$5 = -2k + l \quad (2)$$
$$7 = k + 3l \quad (3)$$

From (1), $l = -3$.
Substitute this in (2).

$$\begin{aligned} 5 &= -2k - 3 \\ -2k &= 5 + 3 \\ k &= -4 \end{aligned}$$

Substitute $l = -3$ and $k = -4$ in (3).

$$\begin{aligned} -4 + 3(-3) &= -4 - 9 \\ &= -13 \\ 7 &\neq -13 \end{aligned}$$

The vectors are noncoplanar.

Topic 4

______ marks